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Reasoning and Computation in Leibniz

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Leibniz's overall view of the relationship between reasoning and computation is discussed on the basis of two broad claims that one finds in his writings, concerning respectively the nature of human reasoning and the possibility of replacing human thinking by a mechanical procedure. A joint examination of these claims enables one to appreciate the wide scope of Leibniz's interests for mechanical procedures, concerning a variety of philosophical themes further developed both in later logical investigations and in methodological contributions to cognitive psychology.

1. Introduction

Leibniz made two broad claims about the relationship between reasoning and computation. The first, and more widely discussed claim was the driving force of the project of *characteristica universalis* (CU): human thinking can be substantially improved with the help of a mechanical procedure guiding our judgements and providing us with a *filum meditandi*. The second claim is directly concerned with the nature of human reasoning: purely combinatorial operations on characters are the only operations involved in human reasoning.[1]

There is a significant connection between these two claims, indicating the opportunity of a joint critical examination. The second claim provides some justification for the project of CU, because it suggests that in Leibniz's view this project was grounded on the idea that reasoning itself involves only combinatorial manipulations of signs; in turn, we argue that the project of CU helps one understanding the scope of Leibniz's claim about the nature of human reasoning: the steps involved in the development of the CU required the use of forms of thought that Leibniz did not characterize as 'mechanical' or 'combinatorial'; accordingly, the claim about the combinatorial nature of human reasoning should not be interpreted as a thesis concerning all human cognitive activities.

On more general grounds, a joint examination of these two claims enables one to appreciate the wide scope of Leibniz's interests for mechanical procedures, concerning a variety of philosophical themes further developed both in later logical investigations and in methodological contributions to contemporary cognitive psychology. Indeed, Leibniz's reflections on mechanical procedures were concerned with the following themes:

(i) analyzing the concept of mechanical procedure (with the aim of isolating epistemologically significant properties of this concept);

1 *PS*, vol. 7, p. 31. For the abbreviations of Leibniz's works, see Part 1 of the bibliography.

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(ii) showing that a suitably chosen mechanical procedure can *replace*, in terms of results, human reasoning in most fields of intellectual investigation;
(iii) justifying this replacement on the basis of epistemological considerations; and
(iv) clarifying to what extent human thought can *itself* be viewed as a purely combinatorial activity.

Many critical discussions of the CU are focused on Leibniz's ideas about theme (ii), often assessing his ambitions on the basis of modern mathematical results on the limitations of pure formalisms in mathematics. However, the other themes must also be taken into account for a balanced evaluation of the historical and conceptual significance of his reflections on mechanical procedures. Indeed, theme (i) was addressed in full generality during the first half of this century (and, according to many, satisfactorily solved) by Turing's (and Post's) analysis of mechanical calculability; (ii) and (iii) were addressed—albeit in a restricted form concerning only logical and mathematical theories—by, e.g., Frege and Hilbert in their foundational programs; theme (iv), and from a somewhat different perspective (iii) also, are currently discussed in methodological contributions to cognitive psychology.

In this paper, we discuss in outline Leibniz's reflections on each of these themes. Section 2 gives a brief sketch of the aims of the CU—emphasizing, in contrast with so many presentations of this idea, Leibniz's disclaimers about the benefits flowing from the realization of his project. These disclaimers point to the difficulty of viewing his project as including the search for an algorithmic procedure enabling one to solve the *decision problems* expressible in the language of the CU, or even the search for an algorithmic method satisfying suitable 'completeness conditions' with the respect to classes of sentences of the CU. Section 3 outlines the fundamental steps involved in the construction of the CU, isolating two main aspects of this description: (a) Leibniz contributed to an analysis of the notion of mechanical procedure by spelling out properties that the envisaged *filum meditandi* of the CU was supposed to satisfy; (b) the development of an 'alphabet of human thoughts', an essential step towards the construction of the CU, required the use of 'intuitive' forms of thought that he did not characterize as mechanical. Section 4 examines his arguments for the distinguished epistemological status of the notion of mechanical procedure, by analyzing his critical observations on Descartes's *Rules for the direction of the mind*. Finally, in section 5, we examine the claim about the combinatorial character of human reasoning, suggesting an interpretation that appears to be consistent with other remarks that Leibniz made about 'intuitive' and 'discursive' forms of thought in general, and about reasoning in particular.

2. Aims of the *Characteristica universalis*

Leibniz formulated a project for developing a *characteristica universalis* already in the essay *De arte combinatoria*, published in 1666 when he was twenty years old. This project, in its essential lines, was never abandoned by Leibniz. He worked intermittently on it throughout his life, and although his ideas on specific features of the CU evolved over the years, he held on to the basic framework of the original project:[2] '. . . I necessarily arrived at this remarkable thought, namely that a kind of

2 Cf. the remarks in Couturat *1901*, 49–50.

alphabet of human thoughts can be worked out and that everything can be discovered and judged by the comparison of the letters of this alphabet and an analysis of the words made from them'.[3] This statement expresses concisely the fundamental steps that had to be undertaken to work out the CU (the simultaneous development of language and calculus), and the goal of this project, namely the realization of an *ars judicandi* and an *ars inveniendi* for an encyclopedic *scientia generalis*.

The idea of developing a method for manipulating the symbolic expressions of an artificial language, and embodying the principles of these *artes* was regarded by Leibniz as a distinguishing feature of the CU: '. . . yet no one has attempted a language or characteristic which includes at once both the arts of discovery and of judgment, that is, one whose signs or characters serve the same purpose that arithmetical signs serve for numbers, and algebraic signs for quantities taken abstractly'.[4] Indeed, the relationship that he attempted to establish between an artificial language and calculation procedures was a distinguishing feature of his project with respect to many contemporary speculations regarding universal languages and mathematical combinatorics.[5] A more detailed description of the sense in which the CU was supposed to provide us with an *ars judicandi* was given, for example, in a letter to the Duke of Hanover:

> Men will find in it a really infallible judge of controversies, because they will always be able to ascertain if it is possible to decide a given problem on the basis of the available knowledge; and if this [condition] cannot be completely satisfied, they will be able in any case to determine what is most likely, just as in arithmetic one can always judge whether it is possible to predict exactly the number somebody else has in mind, on the basis of what that person told us, and often one will be able to say: this must be one of these two, or three, etc., of such numbers, and to set exact limits to the unknown truth. In any case, it is important to know at least whether what we require cannot be found with the available means.[6]

Leibniz's method would have enabled one to know whether any given problem A can be decided on the basis of available knowledge. By applying the method to A,

3 *PS*, vol. 7, 185: '. . . incidi necessario in hanc contemplationem admirandam, quod scilicet excogitari posset quoddam Alphabetum cogitationum humanarum, et quod literarum hujus Alphabeti combinatione et vocabulorum ex ipsis factorum analysi omnia et inveniri et dijudicari possent'. Cf. also *PS*, vol. 7, 516.

4 *PS*, vol. 7, 184. Cf. also pp. 182–183, and *MS*, vol. 7, 187.

5 Cf., e.g., the discussion of several 17th-century projects for universal languages in Cohen *1954*, and Knobloch *1979* for an informative survey of 16th- and 17th-century combinatorial studies. For a discussion of Leibniz's mathematical studies on combinatorics, see Couturat *1901*, 473–500, and Knobloch *1976*. In several writings, Leibniz emphasized the importance of a joint development of language and calculus, speaking of a *Characteristica combinatoria*; cf. *PS*, vol. 7, 10.

6 *PS*, vol. 7, 26: 'Les hommes trouveroient par là un juge des controverses veritablement infallible. Car ils pourroient tousjours conoistre s'il est possible de decider la question par la moyen des connoissances qui leur sont déja données, et lorsqu'il n'est pas possible de se satisfaire entierement, ils pourront tousjours determiner ce qui est le plus vraisemblable. Comme dans l'arithmetique on peut tousjours juger s'il est possible ou non de deviner exactement le nombre que quelque persone a dans la pensée sur ce qu'elle nous en a dit, et souvent on peut dire: ce doit estre l'un de deux ou de trois etc. tels nombres, et prescrire des bornes exacts à la verité inconnue. En tous cas il importe au moins de sçavoir que ce qu'on demande n'est pas trouvable par les moyens que nous avons'.

one of the following outcomes would be forthcoming: either a 'solution' to A, obtained on the basis of the available means, or the answer that A does not admit a solution. In the latter case, however, he also claimed that one might be able to approximate a solution by determining 'what is most likely'.

These qualifications point to the difficulty of identifying *tout court* the rather vague idea of an *ars judicandi* applicable to each problem expressible in the language of the CU with the idea of an algorithmic decision procedure that in principle enables one to answer, either by 'yes' or by 'no', any particular problem belonging to a given class of (mathematical) problems. Indeed, by means of the *ars judicandi* one may also arrive at the conclusion that no such answer to a given problem is obtainable on the basis of the available knowledge. If a comparison with more modern conceptual frameworks is permissible at all, it seems more appropriate, in light of Leibniz's qualifications, to view the idea of an *ars judicandi* as an algorithmic method that would enable one to settle every problem in the same sense in which Hilbert thought that every *mathematical* problem is solvable:

> Occasionally it happens that we seek the solution under insufficient hypotheses or in an incorrect sense, and for this reason we do not succeed. The problem then arises: to show the impossibility of the solution under the given hypotheses, or in the sense contemplated . . . every definite mathematical problem must be susceptible of an exact settlement, either in the form of an actual answer to the question asked, or by a proof of the impossibility of its solution and therewith the necessary failure of all attempts.[7]

In connection with the *ars inveniendi*, Leibniz made similar qualifications, with the aim of preventing the charge of 'hoping or boasting impossible things':[8]

> Thus, by applying the required intelligence, everything—that is, everything that can be obtained from the data by a great and highly trained wit in virtue of reasoning—can be eventually established with an unmistakable method by anybody endowed just with sufficient readiness to act; the strength of this method lying more in acting than in meditating and discovering.[9]

The *ars inveniendi* was thus supposed to provide the means for discovering whatever can be derived from the available data *by a great and highly trained human intellect*. And this particular qualification points to the difficulty of identifying the idea of an *ars inveniendi* with an algorithmic method *complete* at least with respect to some classes of sentences true under their intended interpretation (or logically valid). For example, his statement does not imply that the *ars inveniendi* would have in principle enabled one to derive all the arithmetical sentences expressible in the language of the CU and true under their standard interpretation, unless one supposes that such 'great and highly trained human intellect', solely in virtue of

7 Hilbert *1900*; see Hilbert *1902*, 444.

8 *PS*, vol. 7, 201.

9 *PS*, vol. 7, 202: 'ita enim debita intelligentia adhibita possunt tandem reperiri omnia a quovis et methodo certa, quantum ratione ex datis a maximo etiam ingenio atque exercitatissimo obtineri possent, solo promtitudinis discrimine manente, cujus magis in agendo quam in meditando inveniendoque momentum est'. Cf. *Opuscules*, 431.

reasoning, would be in principle capable of attaining this goal.[10] Given these qualifications, however, Leibniz claimed that the realization of the CU would have provided us with an *efficient*, practically applicable method yielding marvellous results in all fields of human knowledge, including history, medicine, law, military art, and problems of daily life.[11]

3. The construction of the *Characteristica universalis*

In order to achieve these results, the following main steps had to be carried out: firstly, working out an 'alphabet of human thoughts'; and secondly, devising an appropriate method for combining the elements of this alphabet and analyzing the expressions made out of them.[12] Let us consider these tasks in more detail, starting from the problem of working out the alphabet of human thoughts. This alphabet, Leibniz stated, seemingly with a shift from symbols to the ideas that they were supposed to denote,[13] 'is the list of those which are conceived per se, and from whose combination our other ideas are developed'.[14] In other passages he referred to the alphabet of human thoughts as to the list of primitive notions,[15] which are in turn called also 'ideas conceived per se',[16] 'simple' or 'first' notions.[17]

Leibniz argued that there must be primitive notions, for otherwise human understanding would be impossible:

> There must be simple terms, for if we do not conceive of anything per se, we don't conceive of anything at all. It would be as though we should respond to a questioner always using words that he does not understand, and, when he asks for an explication of this, by again using others that he doesn't understand, so that if I keep on in this way you will not understand anything.[18]

10 *PS*, vol. 7, 202. For more detailed discussions of the aims of Leibniz's project, as well as the distinguished roles of the *ars inveniendi* and *judicandi*, see e.g. Couturat *1901*, 178–179; Risse *1969*, 107–116; Hermes *1969*, 92–102; Arndt *1971*, 208, 211; and Danek *1975*, 81.

11 *PS*, vol. 7, 201.

12 *PS*, vol. 7, 185. Leibniz was an optimist about the feasibility of these tasks: with the help of a restricted number of collaborators, he thought that the CU could be fully realized within a few years (cf. *PS*, vol. 7, 187). Elsewhere, however, as we shall see later on, he expressed also some hesitations about the possibility of carrying out the first step.

13 In this connection, Mates *1986*, 8–9 observes: '. . . Leibniz often falls into what is today called "use-mention confusion". Usually, when we ask ourselves whether he is talking about language, thought, or the world, the answer seems to be "none and all of these". . . . We often would very much like to know whether, in saying such and such, the author meant this or that. For example, we ask whether, in the *Categories*, Aristotle was classifying words or things. Part of what he says suggests the former, and part suggests the latter. Probably the truth is that the distinction was below his level of definiteness of intention when he wrote the text in question: he didn't mean words and he didn't mean things; he just wasn't attending to that distinction, though of course he was perfectly capable of drawing it if the matter had been raised. I think that the same considerations apply to Leibniz'.

14 *Opuscules*, 430: 'est catalogus eorum quae per se concipiuntur, et quorum combinatione ceterae ideae nostrae exurgunt'.

15 *Opuscules*, 435.

16 *PS*, vol. 7, 295.

17 *PS*, vol. 7, 293. It seems therefore legitimate to use interchangeably such expressions as 'primitive notion', 'simple' or 'first notion', 'idea conceived per se'. Cf. also *PS*, vol. 4, 452. See, for a characterization of 'idea', *PS*, vol. 7, 263.

18 Quoted in Mates *1986*, 59.

The idea of primitive notion is elaborated on in *De Synthesi et Analysi* with a distinction between confused and distinct primitive notions: 'The first notions, from whose combination the other notions arise, are either distinct or confused; distinct are those that are understood per se, as that of being; confused (and yet clear) are those which are perceived per se, like something coloured, that we cannot explain in any other way except by showing it'.[19]

There are primitive notions, and they must be isolated in order to work out the alphabet of human thoughts. But how do we get to know primitive notions? In *Meditationes de cognitione, veritate et ideis*, Leibniz stated that our knowledge of distinct primitive notions can only be intuitive.[20] Therefore, since there are distinct primitive notions such as that of being (*ens*), and the only form of knowledge of such notions is intuitive, intuitive knowledge must be involved in the process of constructing the alphabet of human thoughts.

It is important to emphasize at this point that intuitive forms of knowledge and thought are contrasted in *Meditationes* with symbolic knowledge and thought.[21] Symbolic thought, also called by Leibniz 'blind thought', is mostly used when dealing with non-primitive notions that are highly complex: if one thinks of a polygon with one thousand equal sides, Leibniz argued, one is often only dimly and imperfectly aware of the ideas corresponding to the linguistic expressions 'side', 'equality' and 'one thousand', and in thinking one makes use of those expressions rather than of the associated ideas. Symbolic and intuitive forms of thought are often intertwined, and in such cases we have to speak of the degrees in which the corresponding knowledge is symbolic or intuitive. But purely intuitive knowledge is required in order to isolate the primitive notions needed for the construction of the alphabet of human thoughts.[22]

So far we have only pointed out that intuitive knowledge is a feature necessarily involved in the processes that are needed for developing the alphabet of human thoughts, without asking whether these processes can actually be carried out by human beings. And Leibniz's observations on this latter question are quite surprising. In contrast with so many optimistic statements about the realizability of the CU that one finds in his writings, he expressed also fundamental doubts about the possibility of isolating all primitive notions, and in turn this seems to jeopardize the possibility of carrying out completely his project:

> Whether the perfect analysis of the notions can ever be accomplished by us, or whether we will be able to reduce our thoughts to the first possibles and to analytical notions, or (what amounts to the same) to the absolute divine

19 *PS*, vol. 7, 293: 'primae notiones quarum combinatione fiunt ceterae aut sunt distinctae aut confusae; distinctae quae per se intelliguntur, ut Ens; confusae (et tamen clarae) quae per se percipiuntur, ut coloratum, quod non possumus alteri explicare nisi monstrando . . .'.

20 *PS*, vol. 4, 423. Intuition has at least two forms in Leibniz: the grasping of a notion that cannot be defined (*Monadologie*, § 35) or the apperception of a complex proposition composed of two or more truths (cf. *PS*, vol. 4, 449–451; *Nouveaux essais*, IV. 2, 367). For the relationship between intuition and the doctrine of innatism, see *PS*, vol. 7, 55 and 111, and *Nouveaux essais, passim*.

21 *PS*, vol. 4, 422–424.

22 *PS*, vol. 4, 423.

attributes themselves, which are indeed the first causes and the ultimate reason of things, I do not dare, however, to define at this moment.[23]

We are thus confronted with a rather strange situation: isolating primitive notions was fundamental for developing an alphabet of human thoughts and thus for carrying out the project of CU, and yet Leibniz expressed doubts about the feasibility of this step.[24]

The second fundamental step of Leibniz's project was the development of a method for combining the signs of the CU and analyzing its complex expressions. He often changed his mind about specific features of the envisaged method, and did not go beyond sketchy and tentative outlines, but he never relinquished a basic requirement: the method had to be cast in the form of a calculation procedure.[25] In his attempts to explain why this restriction was so significant from an epistemological point of view, Leibniz isolated some general features of algorithmic procedures, thus contributing to a conceptual analysis of this notion.

In the first place, Leibniz pointed out that an algorithmic procedure must *determine* completely what actions have to be undertaken by the computing agent. In a letter to Oldenburg, he compared the envisaged method of the CU to parapets placed on both sides of a bridge, preventing one from deviating from the prescribed instructions.[26] Indeed, the CU, once developed, would have provided a mechanical *filum meditandi*.[27]

Secondly, Leibniz emphasized that the instructions of a calculation procedure can be viewed as prescribing *operations on symbolic expressions* in general, and not just on numerical expressions. In a letter to Tschirnhaus he observed: 'A calculation is nothing but operation through characters, and this has its place not only in matters of quantity but in all other reasoning as well'.[28]

Thirdly, Leibniz made also some remarks about the properties of symbolic expressions that play a role in calculation processes. Such processes are arguments *in forma*, he claimed in a letter to G. Wagner:

> Even the addition, multiplication or division of numbers, as one learns these things in school, are forms of proof (*Argumenta in forma*) and one can rely on

23 *PS*, vol. 4, 425: 'An vero unquam ab hominibus perfecta institui possit analysis notionum, sive an ad prima possibilia ac notiones irresolubiles, sive (quod eodem redit) ipsa absoluta Attributa DEI, nempe causas primas atque ultimam rerum rationem, cogitationes suas reducere possint, nunc quidem definire non ausim'. Elsewhere, Leibniz's scepticism concerns the complete list of primitive notions (cf. *Opuscules*, 431). The same point is made by Leibniz in another fragment published by Couturat, *Opuscules*, 511–515.

24 See also, Mugnai *1976*, 92.

25 Cf. *Opuscules*, 85.

26 *PS*, vol. 7, 14.

27 *Ibidem*. To make this point forcefully, Leibniz indulged in a rhetorical exaggeration—oblivious of his own reminders about the possibility of miscalculating—when he claimed that in calculating we cannot make mistakes even if we want to: 'Algebra which we hold in such esteem, is nothing but a part of this general device. Yet it accomplishes this much, that we cannot err even if we wish, and that truth can be grasped as if given in a picture, as if expressed on paper with the aid of a machine. I have come to understand that everything of this kind which algebra proves is nothing but the result of a higher science which I now usually call combinatorial characteristic' (*PS*, vol. 7, 10). For the subordination of algebra to the *Ars Combinatoria*, compare also p. 298, and *MS*, vol. 1, 186; see also Couturat *1969*, 270 and Thiel *1965*, 10.

28 *MS*, vol. 4, 462.

> them, because they prove in virtue of their form. And in this way, one can say that the entire calculation of an accountant is a formal inference and consists of arguments *in forma*. The same is true of Algebra and many other formal proofs, i.e. they are bare and yet perfect.[29]

Thus, only physical properties of symbols—such as their shape and arrangement—and not, e.g., their meaning, play a role in a calculation process.

Fourthly, only elementary intellectual capabilities are required on the part of the executor of a calculation procedure, just because in a calculation process one has to take into account only the shape and arrangement of the symbols and not their 'semantic content', e.g., the ideas that they may express: 'I call *filum meditandi* a sensible and, as it were, mechanical direction of the mind which everybody, even the most stupid ones, can recognize'.[30] Indeed, if calculations are arguments *in forma*, only the capability of discriminating perceptually between symbols and combinatorially manipulating them is required of a computing agent.

4. Computation and epistemology

Leibniz emphasized the epistemological significance of the envisaged *filum meditandi* in his critical remarks about Descartes's *Rules for the direction of the mind.* On several occasions, he pointed to the lack of adequate criteria for 'true knowledge' in Descartes,[31] and claimed that he did not provide rules of reasoning but only too general and vague guidelines that required considerable intellectual efforts and insight to be complied with. The CU, on the contrary, was supposed to enable really everybody to make progress in most fields with 'determinate reason':

> Arranging everything in proper order, refraining from admitting as certain anything except what is clear and distinct, dividing difficulties into parts, keeping the middle course, taking into account the goals, following reason: these are the philosopher's precepts. Excellent in themselves, they can be followed only by great men, more because of their nature and education, than in virtue of the strength of the method. In contrast, the *filum meditandi*, once given, will enable us to make progress in most things with determinate reason, so that men will be made free from a great anxiety and will be given what usually torments them.[32]

The properties of mechanical procedures discussed in the previous section play a crucial role in Leibniz's arguments for this claim. The envisaged rules of the CU are not vague and general suggestions, as they determine completely the actions of the computing agent: the executor of a mechanical procedure conducts his reasoning as if guided by Ariadne's thread (*filum Ariadnaeum*). No insight and little intellectual efforts are required, as the CU relieves us completely of the need of reflecting on the

29 *PS*, vol. 7, 519: 'ja selbst *addition*en, *multiplication*en, oder *division*en der zahlen wie man sie in den Rechenschulen lehret, sind beweissformen (*Argumenta in forma*) und man kan sich darauff verlassen, weil sie krafft ihrer form beweisen. Und auff solche weise kan man sagen dass eine ganze buchhalters rechnung förmlich schliesse, und aus *Argumentis in forma* bestehe. So ist es auch mit der *Algebra* und vielen andern förmlichen beweisen bewand, so nehmlich nackend und doch vollkommen'.

30 *PS*, vol. 7, 14; cf. also 202 and *MS*, vol. 4, 482.

31 *PS*, vol. 1, 384; *PS*, vol. 4, 328 and 425.

32 *PS*, vol. 7, 14.

ideas associated to characters. By 'discharging our imagination'[33] the CU provides a marvellous help to thinking: it is impossible to observe clearly in our mind all ideas associated to characters, and if one had to, our reasoning would be greatly hindered.[34] Not even the memory of those who follow the rules of the CU is involved in the computation process: the use of written signs relieves our memory.[35] Errors in the application of the rules can be easily detected by anybody, in the same way as one checks whether an error has been made in executing an arithmetical operation.[36] In short, the application of the rules involves only consideration of concrete objects (characters) and the capability of combinatorially manipulating them—but no higher, more complicated or more subjective mental processes that, on the contrary, are required in the application of Descartes's rules.[37]

Leibniz's observations comparing the CU with Descartes's method introduce distinctions concerning various grounds upon which our judgments rest and thus address the very problem with which Descartes was concerned when he formulated his *Rules for the direction of the mind.* In general, these observations are not concerned with the more radical, sceptical problem raised by the hyperbolic doubt, which Leibniz deemed to be an improbable hypothesis.[38] An exception is a passage quoted by Couturat. Leibniz presented there an argument challenging the methodic doubt, which is based on evident properties of mechanical procedures: by appealing to the possibility of avoiding the memorization of previously derived sentences (as well as of computation rules) he attempted to undermine the starting point in Descartes's considerations about human knowledge. Leibniz pointed out that, according to Descartes, not even proofs are immune from the tricks of a *malin génie*: each step of a proof relies on the memory of assumptions and previously derived propositions, and an evil genius might well deceive our memory. However, if one follows the CU, written signs relieve us of the necessity of relying on our memory: both the rules of the method and the sentences entered in a proof can be recorded on paper. Thus, Leibniz claimed, if we follow the CU no evil genius is given the possibility of deceiving us.[39]

33 *Nouveaux essais*, IV. 17, 488–489.

34 *PS*, vol. 7, 204.

35 Cf. the quotation from Leibniz in Couturat *1901*, 95 n., reprinted here in footnote 39.

36 *PS*, vol. 7, 205. Frege's epistemologically motivated use of algorithmic procedures in his foundational program was based exactly on this property. Indeed, the need for a reliable method enabling one to detect each assumption of a mathematical proof led Frege to develop the logical calculus of the *Begriffsschrift* as a tool for his logicist program in the foundations of mathematics. Frege maintained, and wanted to establish beyond doubt that arithmetic (which for him included also the theory of real numbers) was analytical, in the sense that the proof of true arithmetical statements required only the assumption of purely logical axioms. In order to achieve this goal, each hypothesis in the proof of arithmetical statements had to be clearly isolated. The *Begriffsschrift*, in view of the algorithmic character of its rules of inference, enabled everybody 'to test the conclusiveness of a chain of inferences in a most reliable way, and to point out every presupposition that tries to sneak in unnoticed, so that its origin can be investigated' (Frege *1879*, x).

37 Notice that the same feature of mechanical procedures is appealed to in recent arguments suggesting that a *computational* theory of higher cognitive processes would not be affected by the problem of circularity or infinite regress, because it would explain such higher cognitive processes in terms of mechanical procedures requiring for their execution only a *modicum* of intellectual capabilities. See Tamburrini *1989* for a critical discussion of an argument supporting this view.

38 *PS*, vol. 4, p. 329.

39 Cf. Couturat *1901*, 95n: 'Conscientia est nostrarum actionum memoria. Cartesius vult ideo nulli demonstrationi posse fidi, quia omnes demonstratio memoria praecedentium propositionum

This argument is insufficient to meet Descartes's sceptical challenge for various reasons. For example, perceptual capabilities are required in executing a mechanical procedure, and these could be altogether generated or distorted by a *malin génie*, if we admit its existence. Furthermore, the construction of the CU involves the development of a calculus and of an alphabet of human thoughts, and the *malin génie* could also trick us during this preliminary design stage.

This argument contributes to point out the epistemologically motivated use of mechanical procedures in Leibniz; but, as we argued, it is inconclusive and, moreover one can hardly see its point in the overall economy of Leibniz's philosophy: just as Descartes, he appealed to God in epistemological matters, at least when he provided a foundation for our knowledge of the external world by introducing the notion of pre-established harmony.

Leibniz's observations about the relationship between the CU and Descartes's *Rules* are focused on the problem of comparing methods and selecting the more reliable ones. In this respect, the CU was supposed to be a real improvement over Descartes's method, transferring the same confidence we have in mathematical proofs to the results obtainable in other domains. That this was the main concern of Leibniz's epistemological considerations about the CU emerges clearly from his comment on Descartes's thesis that also mathematical proofs can be doubted:

> There can be no doubt in mathematical demonstrations except insofar as we need to guard against error in our arithmetical calculations. For this there is no remedy except to re-examine the calculation frequently or to have it tested by others and also to add confirmatory proofs. This weakness of the human mind arises from a lack of attention and memory and cannot be completely overcome, and Descartes's mention of it, as if he knew a remedy, is in vain. It would be enough if the state of affairs in other fields were the same as that in mathematics; indeed, all reasoning, even the Cartesian, however convincing and accurate, is subject to this doubt, whatever may be said about some powerful deceiving spirit or about the distinction between dreams and waking.[40]

In concluding this section, we note in passing that another central charge of Leibniz against Descartes's *Rules* concerned the characterization of the intuitive, non-discursive basis of our knowledge, that we have examined from another perspective in section 3. In particular, Leibniz criticized the absence in Descartes of adequate criteria for clear and distinct perceptions.[41] Human knowledge for Leibniz was founded on elementary truths, which are known not only as clear and distinct, but also as primary and unprovable.[42] Certainty about them is based on the impossibility of analyzing them in still more elementary truths. Descartes's clear and

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indiget; in qua nos potentia alicujus mali genii fortasse falli(?) posset. Sed si hucusque producimus dubitandi titulos, etiam conscientiae nostrae de praesentibus fidere non licet. Semper enim involvitur memoria, cum nihil sit absolute loquendo praesens praeter momentum. Memoriam in demonstrando sublevant scripturae seu notae, nullum autem dari malum genium, qui nos in illis quam(?) adulterandis fallat'.

40 *PS*, vol. 4, 356. Loemker *1969*, 384.

41 *PS*, vol. 1, 384; *PS*, vol. 4, 274 and 331. For a discussion of this problem, see also Couturat *1901*, 202; Belaval *1960*, 133 *f.*; and Nador *1965*, 144–157.

42 *Nouveaux essais*, IV. 7, 406 f; cf. also Schulz *1970*.

distinct perception is thus to be replaced by some kind of 'immédiation entre le sujet et la prédicament'.[43]

5. Reasoning as computation

Aside from suggesting that human thinking can be substantially improved by a mechanical procedure operating on the symbolic expressions of the CU, Leibniz formulated also the thesis that human reasoning (*ratiocinatio*) involves only the execution of purely combinatorial operations: 'All our reasoning is nothing but connection and substitution of characters, whether these characters are words, marks, or finally images'.[44]

There is thus a substantial homogeneity between the operations characterizing human reasoning and the envisaged rules of the CU: both of them involve, as Leibniz already emphasized in *De arte combinatoria*, combinatorial manipulations of characters. But what is the place of reasoning in human thinking? In section 3 we argued that an intuitive form of thought which Leibniz did not characterize as mechanical or combinatorial—indeed contrasted with *symbolic* or *blind* thought—was involved in the construction of the CU. This suggests that the thesis about the combinatorial character of human reasoning cannot be interpreted as a thesis about all human cognitive activities.

The same point is also suggested by a 'taxonomic' observation of Leibniz about human reasoning. Reasoning is a human activity that distinguishes man from the other animals and from God.[45] Animals have perceptions, which are a form of representation shared also by human beings. But this is not a conscious form of representation, and only when a representation is accompanied by consciousness we call it thought.[46] Reasoning, in turn, is a particular component of thought; thought includes also intuition, shared by human beings, but only to a certain extent, with supernatural beings.[47]

If reasoning does not coincide with human thought, what is its specific role in human cognition? In the *New essays*, Philalèthe provides the following characterization of reasoning, which is not challenged by Théophile:

43 *Nouveaux essais*, IV. 9, 434.

44 *PS*, vol. 7, 31: 'Omnis Ratiocinatio nostra nihil aliud est quam characterum connexio et substitutio, sive illi characteres sint verba, sive notae, sive denique imagines'. See also Hobbes's statement quoted in *PS*, vol. 4, 64. The view that reasoning is a kind of calculation was a well established element of the Ramist tradition. The comparison of a syllogism with a form of calculation is found in many Renaissance authors after Peter Ramus (1515–1572) (see Nuchelmans *1980*, 169, note 3; see also Gassendi *1658*, vol. 1, 106a). That this idea is so often regarded as originating with Hobbes is perhaps partly due to the fact that Leibniz refers to him in *De arte combinatoria*. For Leibniz's relation to Hobbes's ideas: Couturat *1969*, 457–472; Mittelstrass *1970*, 430–431; and Danek *1975*, 85f. Dascal *1976*, 21*f* charges Couturat of belittling Hobbes's influence on Leibniz. See also Heinekamp *1972* and Mugnai *1973*.

45 *PS*, vol. 7, 530 and 331.

46 *PS*, vol. 2, 112. As a matter of fact, the continuity of perceptions constituting any existing soul and expressing present, past and future states of the universe, has different levels. In the animals this continuity is based on "similar" responses to similar or corresponding sense-impressions, which occur according to the laws of association. But association is not to be confused with reasoning. Only man is capable of reasoning; reasoning presupposes intuition, forms of which were theorized by Leibniz in the *Nouveaux essais* as apperception and reflection, i.e. the mind's awareness of its own processes.

47 Cf. McRae *1976*, chs. 3 and 4.

> Demonstrative knowledge is only the stringing together of intuitions in all the connections of intermediary ideas, because often the mind is unable to join, compare or apply immediately ideas to each other. This forces one to use other (one or more) intermediate ideas to discover the agreement or disagreement one is searching for, and this is what is called reasoning.[48]

Reasoning is here described as a process by which one constructs a sequence or chain of ideas. This chain enables us to discover the 'agreement' or 'disagreement' of ideas when these cannot be immediately compared with one another. A demonstration founded on intermediate ideas provides us with an example of knowledge obtained by reasoning.

Leibniz also emphasized that characters are a fundamental ingredient of reasoning. Indeed, he claimed that signs are necessary to human reasoning,[49] and that 'human reasoning is carried out by means of some kind of signs or characters'.[50] But how can one step from the necessity of using characters in reasoning to the thesis that reasoning *is* just a combinatorial manipulation of signs, without contradicting Leibniz's statement that reasoning is supposed to construct a chain of ideas? It is not easy to answer this question, but one may advance a conjecture that seems consistent with the observations that we have so far examined, and that enables one to relate this thesis to the project of CU. Let us consider Leibniz's remarks about the relationship between characters and ideas:[51]

> Indeed, even if the characters are arbitrary, their use and connection has something that is not arbitrary, namely some kind of proportion between them and the things, and the mutual relationships of different characters that express the same things. And this proportion or relation is the foundation of truth. This has the consequence that whether we use these or other characters, the same result or an equivalent one, or one corresponding in proportion will always obtain.[52]

This widely discussed claim about a form of correspondence between characters and ideas—that is only imperfectly present in natural languages—suggests that the construction of a chain of characters, by purely combinatorial operations, may manifest in the concrete relationship between characters the relationship holding between the corresponding ideas.[53] Since this latter relationship can be read off

48 *Nouveaux essais*, IV. 2, 367: 'Or la connoissance démonstrative n'est qu'un enchainement des connoissances intuitives dans toutes les connexions des idées mediats. Car souvant l'esprit ne peut joindre, comparer ou appliquer immédiatement les idées l'une à l'autre, ce qui oblige de se servir d'autres idées moyennes (une ou plusieurs) pour découvrir la convenance ou disconvenance qu'on cherche, et c'est ce qu'on appelle raissonner'.

49 *PS*, vol. 7, 191.

50 *PS*, vol. 7, 204.

51 Cf. on this point the informative discussion in Mugnai *1973*.

52 *PS*, vol. 7, 192: 'Nam esti characteres sint arbitrarii, eorum tamen usus et connexio habet quiddam quod non est arbitrarium, scilicet proportionem quandam inter characteres et res, et diversorum characterum easdem res exprimentium relationes inter se. Et haec proportio sive relatio est fundamentum veritatis. Efficit enim, ut sive hos sive alios characteres adhibeamus, idem semper sive aequivalens seu proportione respondens prodeat'. Cf. also *PS*, vol. 7, 263–264: 'et quod expressionibus istis commune est, ex sola contemplatione habitudinum experimentis possumus venire in cognitionem proprietarum respondentium rei exprimendae'.

53 Cf. Heinekamp *1972*, 464–469.

from the former, it seems reasonable to assert that a purely combinatorial manipulation of signs may provide exactly what reasoning is supposed to provide: a chain of ideas.

Leibniz stated that human mental activity is governed by general laws which, according to the doctrines of monadology and pre-established harmony, are immune from any 'external correction'; the human soul, just because it conforms to such laws, can be regarded as an immaterial 'automaton' or 'machine'.[54] His thesis about the nature of reasoning might be viewed as a principle concerning the functioning of one specific aspect of the human soul. This functioning, and the working of human thinking as a whole, which includes but does not coincide with reasoning, are not entirely satisfactory—due, e.g., to the imperfect 'correspondence' between the characters of ordinary languages and our ideas, or to the difficulty of entertaining clearly, analyzing and connecting ideas in our minds. But these defects were to be at least partially rectified by the project of CU. In our mental activity, intuitive thinking (by which we entertain, compare, and analyze *ideas*) and reasoning (by which we operate on the *characters* corresponding to those ideas) are usually intertwined.[55] With the project of CU, Leibniz envisaged the possibility of improving our thinking, exploiting both aspects that are present in in our mental activity, but assigning them sharply distinct roles. Intuitive thinking was crucial for designing the CU. But once the project was carried out, the resulting algorithmic method for discovering and judging would have only involved the combinatorial operations that characterize human reasoning. This conceptual framework constitutes the chief invariant aspect of Leibniz's attempts at developing a CU, and its initial presentation can be found already in *De arte combinatoria*. Indeed, he argued there that once the list of all primitive terms had been isolated and each of them was associated with an arithmetical sign, the art of discovering complex notions and truths would have involved only combinatorial manipulations of these signs.

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54 *PS*, vol. 4, 522. *PS*, vol. 4, 548–549; *Monadologie*, § 18. See also *PS*, vol. 4, 558, 532–533; *Monadologie*, § 79; and McRae *1976*, 47–48.

55 Cf. the remarks in *Meditationes de cognitione, veritate et ideis* in *PS*, vol. 4, 423.

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